Performance Analysis of Full Duplex and Selective and Incremental Half Duplex Relaying Schemes

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Abstract—In this work we compare full-duplex (FD) and half-duplex (HD) relaying in terms of outage probability and throughput. We consider a practical FD relay model where the loop interference between transmitted and received signals is taken into account. We analyze two modes for FD transmission, block Markov encoding and multi-hop transmission without interference cancellation, and two different modes for HD transmission, which are based on selective and incremental decode-and-forward protocols. Results show that there is a tradeoff between SNR and information rate in which each scheme becomes more suitable.

I. INTRODUCTION

Cooperative communication is an alternative to achieve spatial diversity even with single antenna devices, as shown in [1]. Probably the most known cooperative protocols are the amplify-and-forward (AF) and the decode-and-forward (DF) [1], including their variants selective and incremental. Moreover, the cooperative protocols can operate either in a half-duplex (HD) or full-duplex (FD) fashion.

In the selective-DF (SDF) protocol, if the relay was able to decode the source message then it always cooperates forwarding that message to the destination. When implemented using HD devices the SDF protocol suffers from a multiplexing loss2, since the relay has to listen to the source in the first time/frequency slot and then forward the message in a second slot [1]. Therefore, a second slot is always allocated to the relay operation. By its turn, incremental cooperative protocols, such as incremental-DF (IDF), can overcome the spectral inefficiency of HD cooperation through the exploitation of a return channel between nodes [1], [4]. In the IDF protocol the relay only cooperates with the source if a retransmission is requested by the destination, so that it is not necessary to previously allocate a time slot for the relay operation.

Moreover, FD cooperative relaying in general achieves higher capacity than HD cooperative protocols, as shown in [4]. The best known performance achieving FD scheme is the block Markov (BM) encoding [4], [5]. The authors in [5] analyze the FD-BM technique and derive the outage probability of an ideal (without loop interference) cooperative FD-BM scheme. However, perfect isolation between transmitted and received signals is often not possible. In practice, isolating

1Several works have been proposed to recover some of the spectral loss inherent to the HD cooperative schemes, as [2], [3] and references therein, but in this paper we focus only on the regular SDF protocol.

the transmitted and received signals is not straightforward, once the transmitted power is normally much larger than the received power [4]. Practical non-cooperative FD multi-hop (FD-MH) relaying models were proposed in [6]–[9], where a power leakage between transmitted and received signals is assumed, also known as loop interference. For instance, in [6] it is shown that practical non-cooperative FD-MH relaying is feasible even if the relay faces strong loop interference, and that non-cooperative FD-MH relaying enhances capacity when compared to non-cooperative multi-hop HD relaying. Similar conclusions were obtained in [7]–[9].

In this paper we consider a practical FD relay, with loop interference, which may operate under: i) cooperative FD-BM, which is a high-performance and high-complexity approach; ii) non-cooperative FD-MH, which is less performing and simpler than FD-BM. Differently than previous works, we compare the performance of FD relaying to that of selective and incremental HD relaying protocols using incremental redundancy (IR). In IR cooperative schemes, instead of just retransmitting the source message, the relay sends additional parity bits to the destination, which are then combined by the destination with the first source transmission. Cooperative schemes based on the IR principle, also known as parallel coding, outperform those based on the simple retransmission of the source message [10]. The main contribution of this paper is to show that there is a tradeoff between SNR and the attempted information rate in which HD schemes become more attractive than FD schemes. In addition, we show that the performance of HD relaying is even more competitive when an incremental protocol is considered. These conclusions hold even when power and rate allocation are carried out.

The rest of this paper is organized as follows. Section II presents the system model. Section III introduces the outage and throughput analysis. Section IV presents some numerical results, while Section V concludes the paper.

II. SYSTEM MODEL

Consider a system with three terminals: source (S), relay (R) and destination (D), as in Fig 1. The S-D, R-D, and S-R channels are all subject to quasi-static Rayleigh fading, with zero mean and unit variance. We assume perfect channel state information at the receivers. The noise is of the complex

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additive white Gaussian noise type with variance $N_0/2$ per dimension. Throughout this paper we assume that $N_0 = 1$.

In FD mode, $S$ broadcasts the message $x_S$, which is heard by both $D$ and $R$. At the same time $R$ sends a message $x_R$ to $D$. Moreover, since we assume the presence of a loop interference, the transmission from $R$ interferes itself in the reception at $R$. Then, the received signals at $R$ and at $D$ can be written as:

$$y_R = \sqrt{P_S \kappa_{SR}} h_{SR} x_S + \sqrt{P_R h_{RR}} x_R + n_R,$$

$$y_D = \sqrt{P_R \kappa_{RD}} h_{RD} x_R + \sqrt{P_S h_{SD}} h_{DS} x_S + n_D,$$

where $h_{SD}$, $h_{SR}$ and $h_{RD}$ are the complex fading channel coefficients of the S-D, S-R and R-D links, respectively, while $h_{RR}$ is the complex fading coefficient of the loop interference [9]. The terms $n_R$ and $n_D$ are the noise at $R$ and $D$, while $\kappa_{ij}$ is the path loss between nodes $i$ and $j$. The actual construction of the source message $x_S$ is a function of the information to be transmitted and of the FD scheme in use (FD-BM or FD-MH).

In the HD mode the transmissions are orthogonal in time. First, $S$ broadcasts a message to $R$ and $D$, so that:

$$y_{SR} = \sqrt{P_S \kappa_{SR}} h_{SR} x_S + n_R,$$

$$y_{SD} = \sqrt{P_S \kappa_{SD}} h_{SD} x_S + n_D.$$  

When $R$ cooperates, in a second time slot $D$ receives the signal:

$$y_{RD} = \sqrt{P_R \kappa_{RD}} h_{RD} x_R + n_D,$$

where $x_R$ is the decoded message at $R$. Then, $D$ combines the signals from $S$ and $R$. Note that $R$ only forwards the message if it is free of errors, according to the SDF protocol. Moreover, if a return channel is available and the IDF protocol is employed, $S$ and $R$ only forwards the message if it is free of errors and if $D$ requests a retransmission.

### III. OUTAGE AND THROUGHPUT ANALYSIS

#### A. Multi-Hop Full-Duplex Relaying

In the non-cooperative FD-MH mode, $S$ broadcasts a message and $R$ forwards it to $D$. The transmission from $S$ is actually seen as interference at $D$. Thus, the instantaneous SNR at $R$ and $D$ are given by:

$$\gamma_R = \frac{|h_{SR}|^2 P_S \kappa_{SR}}{|h_{RR}|^2 P_R + 1}, \quad \gamma_D = \frac{|h_{RD}|^2 P_R \kappa_{RD}}{|h_{SD}|^2 P_S \kappa_{SD} + 1},$$

where $P_S$ and $P_R$ are the source and relay transmit power, respectively. Note the effect of the loop interference in the denominator of $\gamma_R$.

An outage occurs when $I_{ij} < \mathcal{R}$, where $I_{ij}$ is the mutual information in the $i$ to $j$ link, and $\mathcal{R}$ is the attempted information rate. Supposing complex Gaussian inputs and unitary bandwidth [11], the mutual information of the S-R and R-D links, respectively, are given by:

$$I_{SR} = \log_2 (1 + \gamma_R), \quad I_{RD} = \log_2 (1 + \gamma_D).$$  

The overall outage probability of the FD-MH mode is:

$$P_{\text{FD-MH}} = \Pr \{ I_{SR} < \mathcal{R} \} + \Pr \{ I_{SR} > \mathcal{R} \} \Pr \{ I_{RD} < \mathcal{R} \} = P_{SR} + (1 - P_{SR}) P_{RD}.$$  

Suppose the nodes use the same transmit power $P = P_S = P_R$. Then, when $P$ goes to infinity, there is an error floor in the outage probability given by:

$$\lim_{P \to \infty} P_{\text{FD-MH}} = 1 - \frac{\kappa_{SR} \kappa_{SD}}{\kappa_{SR} \kappa_{SD} + (2^\mathcal{R} - 1) \kappa_{RR} \kappa_{RD}} \times \frac{\kappa_{RR} \kappa_{RD} \kappa_{RD}}{\kappa_{RR} \kappa_{RD} P_S \kappa_{SD}}.$$  

The throughput, which is the average information rate seen at $D$, can be written as:

$$I_{\text{FD-MH}} = \mathcal{R} (1 - P_{\text{FD-MH}}).$$  

#### B. Block Markov Full-Duplex Relaying

The capacity for the relay channel is still an open problem. In view of this unanswered issue, the best achievable rate known in the literature is obtained when the BM technique [4], [5], [12] is employed. The FD-BM mutual information is:

$$I_{\text{FD-BM}} = \min (I_{SR,BM}, I_{MAC})$$  

where the mutual information $I_{SR,BM}$ is given by [5]:

$$I_{SR,BM} = \log_2 (1 + (1 - \rho^2) \gamma_R)$$  

while the variable $\rho$ is the correlation coefficient between the source and the relay messages [5], [12]. In fact, $\rho$ can be
chosen in order to maximize $I_{FD,BM}$. The mutual information of the multiple access channel (MAC) formed by the R-D and S-D links is [5], [13]:

$$I_{MAC} = \log_2 \left( 1 + P_S \kappa_{SD} |h_{SD}|^2 + P_R \kappa_{RD} |h_{RD}|^2 + 2 \sqrt{P_S \kappa_{SD} P_R \kappa_{RD}} \Re (p_{SD} h_{RD}) \right), \quad (16)$$

with $\Re(.)$ denoting the real part and $(.)^*$ the complex conjugate. The overall outage probability can be written as [5]:

$$\mathcal{P}_{FD,BM} = \Pr \{ \min (I_{SR,BM}, I_{MAC}) \leq \mathcal{R} \} = 1 - P_{MAC}^C \mathcal{P}_{SR,BM}^C, \quad (17)$$

and the complementary outage probability between S-R is:

$$\mathcal{P}_{SR,BM}^C = \frac{\exp \left( - \frac{2^\gamma - 1}{P_S \kappa_{SR} \pi_{SR} (1 - \rho^2)} \right) P_S \kappa_{SR} \pi_{SR}}{P_S \kappa_{SR} \pi_{SR} + \frac{(2^\gamma - 1)}{(1 - \rho^2)} P_R \kappa_{RR}}, \quad (18)$$

where we considered the effect of the loop interference at the relay (by means of parameter $\pi_{RR}$). The complementary outage probability of the MAC channel is [12]

$$\mathcal{P}_{MAC}^C = \frac{ae^{-\frac{2^\gamma - 1}{\alpha - \beta} \pi_{SR} (1 - \rho^2)}}{\alpha - \beta}, \quad (19)$$

where $\alpha$ and $\beta$ are, respectively, given by:

$$\alpha = \frac{a}{2} + \sqrt{b}, \quad \beta = \frac{a}{2} - \sqrt{b}, \quad (20)$$

and

$$a = (P_R \kappa_{RD} \pi_{RD} + P_S \kappa_{SD} \pi_{SD}), \quad (21)$$

$$b = \frac{a^2}{4} - P_R \kappa_{RD} P_S \kappa_{SD} \pi_{RR} \pi_{SD} (1 - \rho^2), \quad (22)$$

where $\pi_{SD} = E \left[ |h_{SD}|^2 \right]$ and $\pi_{RD} = E \left[ |h_{RD}|^2 \right]$.

Again, supposing equal power allocation $P_R = P_S = P$, then the error floor when $P$ goes to infinity is given by:

$$\lim_{P \to \infty} \mathcal{P}_{FD,BM} = \frac{(2^\mathcal{R} - 1) \pi_{RR}}{(2^\gamma - 1) \pi_{RD} + \kappa_{SD} \kappa_{SR} (1 - \rho^2)} \cdot \quad (23)$$

In the absence of loop interference ($\pi_{RR} = 0$), the error floor of FD-BM is zero.

Finally, the throughput for FD-BM is:

$$\mathcal{J}_{FD,BM} = \mathcal{R} (1 - \mathcal{P}_{FD,BM}). \quad (24)$$

C. IR Half-Duplex Relaying

In the IR half-duplex relaying the retransmission carries new coded bits, so that the receiver can concatenate the originally received packet and the retransmitted packet to form a single packet of lower rate. In that case, after the retransmissions, what we have in the receiver is an accumulation of mutual information [10], [11], [14]:

$$I_{IR} = \log_2 (1 + \gamma_{SD}) + \log_2 (1 + \gamma_{RD}) = \log_2 ((1 + \gamma_{SD})(1 + \gamma_{RD})), \quad (25)$$

where $\gamma_{ij}$ is the instantaneous SNR of the link between the nodes $i \in \{S, R\}$ and $j \in \{R, D\}$, which is given by $\gamma_{ij} = |h_{ij}|^2 P_i \kappa_{ij}$, while the average SNR is $\bar{\gamma}_{ij} = P_i \kappa_{ij} \pi_{ij}$. By defining two new variables $Z_1 = 1 + \gamma_{SD}$ and $Z_2 = 1 + \gamma_{RD}$, and computing the distribution of the product $Z_1 Z_2$ [15], it is possible to write the outage probability as:

$$\mathcal{P}_{IR} = \Pr \{ I_{IR} < \mathcal{R} \} = \Pr \{ Z_1 Z_2 < 2^\mathcal{R} \}$$

$$= \frac{1}{\gamma_{SD} \gamma_{RD}} \int_1^{2^{\mathcal{R}}} \int_1^{\infty} \frac{1}{w} \exp \left( \frac{1}{\gamma_{SD}} - \frac{1}{\gamma_{RD}} \right) dw dz. \quad (26)$$

The throughput is determined based whether a return channel is available or not. Moreover, we assume that, if the relay could not decode the source message, then neither the relay nor the source retransmit in the second time slot.

1) IR-SDF Half-Duplex Relaying: The scheme is based on the traditional SDF protocol, which is spectrally inefficient due to the half-duplex constraint. We recall that in the SDF protocol R always retransmits as long as the source message could be decoded. Therefore, a second time slot is always previously allocated to R, which limits the maximal throughput achieved by the IR-SDF scheme in $\frac{\mathcal{R}}{2}$. The throughput for IR-SDF can be written as:

$$\mathcal{T}_{IR,SDF} = \frac{R}{2} (1 - \mathcal{P}_{SD}) + \frac{R}{2} \mathcal{P}_{SD} (1 - \mathcal{P}_{SR}) \left( 1 - \frac{\mathcal{P}_{IR}}{\mathcal{P}_{SD}} \right), \quad (27)$$

Note that $\frac{\mathcal{P}_{IR}}{\mathcal{P}_{SD}} = \Pr \{ I_{IR} < \mathcal{R} | I_{SD} < \mathcal{R} \}$ is the probability that an error occurs at D after the transmission from R, given that an error occurred after the original transmission from S.

2) IR-IDF Half-Duplex Relaying: This is an incremental-relaying protocol, which means that a return channel is available between nodes. Therefore, the destination requires a retransmission from R only when its is necessary. Therefore, the performance, in terms of throughput, is limited by the attempted information rate $\mathcal{R}$. Thus, the throughput for IR-IDF is given by:

$$\mathcal{T}_{IR,IDF} = R (1 - \mathcal{P}_{SD}) + \frac{R^2}{2} \mathcal{P}_{SD} (1 - \mathcal{P}_{SR}) \left( 1 - \frac{\mathcal{P}_{IR}}{\mathcal{P}_{SD}} \right). \quad (28)$$

IV. NUMERICAL RESULTS

We assume a log-distance path loss model with decay exponent 4, and that the transmit power of S and R are the same, $P_S = P_R$, at least when power allocation is not carried out. We also suppose that R is in a straight line between S and D. Normalizing the distance $d_{SR}$ between S and D to the unit, the distance between R and D is $d_{RD} = 1 - d_{SR}$, where $d_{SR}$ is the S to R distance. In the results we considered $d_{SR} = 0.5$. Based on [9], we consider two levels of loop interference: the ideal case, in which $\pi_{RR} = 0$; and the case of $\pi_{RR} = -8$ dB. The correlation coefficient for FD-BM is $\rho = 0$, since the conclusions for this value of $\rho$ do not vary for $\rho \neq 0$. Moreover, $\pi_{SD} = \pi_{SR} = \pi_{RD} = 1$.

Fig. 2 compares the throughput of FD and HD schemes as a function of $\gamma_{SD}$ when $\mathcal{R} = 2$ bits/s/Hz. The FD-BM scheme can considerably outperform both HD schemes, even in the presence of loop interference. For this particular $\mathcal{R}$, the simplest FD scheme analyzed in this paper (FD-MH) is able to outperform the IR-SDF scheme in the whole SNR range.
range, while it outperforms IR-IDF in low to medium SNR. However, for higher values of $\mathcal{R}$ and for practical values of the loop interference the performance of FD relaying considerably decreases as we can see from Fig. 3. For $\mathcal{R} = 8$ bits/s/Hz and $\pi_{RR} = -8$ dB both HD schemes outperform the FD schemes. The performance of the simple FD-MH scheme can be considerably worse than that of the other methods.

In Fig. 4 we analyze the throughput as a function of $\gamma_{SD}$ for a fixed $\mathcal{R} = 0$ dB. At low SNR and with low-medium information rates, the FD schemes outperform both HD strategies. On the other hand, at a high SNR, as show in Fig. 5 where $\gamma_{SD} = 20$ dB, IR-IDF becomes a good approach, presenting a better performance than FD-BM with loop interference. Thus, the FD strategies are a better approach at low-medium SNR regions, while HD schemes are more suitable to high SNR regions, given that the outage probability of HD schemes achieves very small values at high SNR. Notice that from (12) and (23) the error floor is close to the unity for large $\mathcal{R}$, so that the outage probability of the FD schemes increases rapidly at high information rates.

### A. Power and Rate Allocation

We investigate the impact of power and rate allocation (denoted as PA and RA) between S and R. The choice of power and rate is such that it maximizes the throughput. The problem can be formalized as:

$$\max_{P_S^*, P_R^*} \mathcal{T}$$

subject to

$$P_S^* + P_R^* \leq 2P$$

$$\mathcal{R}_{\min} \leq \mathcal{R} \leq \mathcal{R}_{\max}$$

(29)

2The outage probability of a direct transmission from S to D is $P_{SD} = 1 - \exp \left( -\frac{1}{\mathcal{R}_{\min} \gamma_{SD}} \right)$, while the throughput is $\mathcal{T}_{\text{dir}} = \mathcal{R} (1 - P_{SD})$.

where $\mathcal{T}$ can be $\mathcal{T}_{\text{FD-MH}}, \mathcal{T}_{\text{FD-BM}}, \mathcal{T}_{\text{IR-IDF}}, \mathcal{T}_{\text{IR-SDF}},$ or $\mathcal{T}_{\text{dir}}$, and $2P$ is the power used in the direct transmission. The maximization can be performed with respect to $\mathcal{R}$, $P_S^*$, or both. Our goal is to compare the different HD and FD schemes, therefore we do not focus on the proposal of a particular PA and RA solution, instead we resort to numerically efficient algorithms. For RA we considered that $\mathcal{R}$ varies from $\mathcal{R}_{\min} = 1$ to $\mathcal{R}_{\max} = 12$ bits/s/Hz. At each SNR we numerically determine the attempted rate $\mathcal{R}$ which maximizes the throughput. For PA, we determine the values of $P_S^*$, and therefore $P_R^*$ since $P_S^* + P_R^* = 2P$, which maximize the throughput. When PA and RA are carried out at the same time, the two parameters ($P_S^*$ and $\mathcal{R}$) are jointly numerically optimized.

Fig. 6 shows the throughput when PA and RA are jointly...
the direct transmission.

performance of FD-MH, even with PA and RA, is worse than the loop interference caused by the relay transmitted signal it outperforms FD-BM with loop interference. By its turn, the we considered a practical relay model which takes into account competitive, specially from the medium to high SNR region, where performance under power and rate allocation. In the FD schemes we considered a practical relay model which takes into account the loop interference caused by the relay transmitted signal into the relay received signal, while we considered selective and incremental cooperation in the HD case. Our results show that HD schemes, specially of the incremental type, can outperform FD relaying in a number of situations. Moreover, from our results we can conclude that FD relaying becomes an attractive approach to systems that operate at low to medium SNR values, and with low to medium information rates. On the other hand, HD schemes seem to be more suitable to operate at high SNR and high information rate regions. Finally, when power and rate allocation are carried out, IR-IDF presents a very competitive performance with respect to FD-BM with loop interference, being a better option in the high SNR region.

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