Spectrum Sensing of Correlated Subbands With Colored Noise in Cognitive Radios

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Abstract—In this paper, we consider the problem of wideband spectrum sensing by using the correlation among the observation samples in different subbands. The Primary User (PU) signal samples in occupied subbands are assumed to be zero-mean correlated Gaussian random variables and additive noise is modeled as colored zero-mean Gaussian random variables independent of the PU signal. It is also assumed that there is at least a minimum given number of subbands that are vacant of PU signals. First we derive the optimal detector and the Generalized Likelihood Ratio (GLR) detector for the case that the covariance matrix of PUs signal samples is unknown and the noise variance in the different subbands is known. Then, we propose an iterative algorithm for GLR test when both the covariance matrix of the PUs signal samples and the noise variances in the different subbands, are unknown. For analytical performance evaluation, we derive some closed-form expressions for detection and false alarm probabilities of the proposed detectors in low Signal to Noise Ratio (SNR) regime. The simulation results are further presented to compare the performance of the proposed detectors.

I. INTRODUCTION

A main requirement in the Cognitive Radio (CR) networks is spectrum sensing to identify the available spectrum holes. The spectrum sensing must be done in a short time and also in an accurate method by the Secondary User (SU). Several different methods for spectrum sensing have been proposed [1]–[4]. A matched filter can be used when the PU signal and noise variance are both known at the SU [1]. If the PU signal is unknown but noise variance is known at the SU, the Energy Detector (ED)(a.k.a. radiometer) is the optimal method for spectrum sensing [2], [3]. As shown in [5], the performance of the ED degrades in the presence of noise variance uncertainty. In such cases, the SNR must be more than SNR wall to ensure a reliable spectrum sensing. In contrast to the narrowband counterpart, wideband spectrum sensing [6]–[9] relies on the observation samples taken from different subbands which might exhibit some correlation. In [8], it is assumed that observation samples are independent from each other and detection thresholds in each subband are jointly optimized for wideband ED. In [9], the independency assumption is relaxed and the spectrum sensing problem is revisited for correlated subbands. The linear combination of different subband energies is used for detection and the performance of sensing is investigated. In [6], [7], spectrum sensing is applied over multiple subbands when deterministic PU signal and random noise variance are unknown. In [6], Taherpour et al. assume that among the scanned subbands, there is at least a given minimum number of subbands that can be considered vacant of PU signals and the GLR detector is derived for each of such subbands based on this assumption. In [7], the PU signal samples are modeled as zero-mean Gaussian processes that are uncorrelated in different occupied subbands and additive noise is modeled as zero-mean white Gaussian process.

In this paper, we extend the results of [7] to the case that the PU signal samples are correlated in different subbands and variances of noise components are not identical in different subbands. First, we derive optimal detector when the covariance matrix of PUs signal samples and noise variances are known. Then, as practical cases, we derive the GLR detection rules for two cases: In the first case, the covariance matrix of PU signals is unknown, but the noise variance values in the different subbands are known. In the second case, we assume both PU covariance matrix and the noise variances in the different subbands are unknown. For the optimal detector and the first GLR detector case, we obtain closed-form expressions for detection and false alarm probabilities in low SNR regime.

The rest of the paper is organized as follows: In Section II, we describe basic assumptions and the system model. In Section III, we derive the optimal detector. In Section IV, we derive GLR detectors for the cases that covariance matrix of the PU signal is unknown and that both covariance matrix of the PU signal and noise variances are known. In Section V, we evaluate the performance of the optimal detector and proposed GLR detectors analytically. The simulation results and related discusses are given in Section VI and finally, Section VII concludes the paper.

II. BASIC ASSUMPTIONS AND SYSTEM MODEL

Suppose that the wideband spectrum is divided into \( K \) subbands and assume that \( L \) samples are received from each subband for example by using FFT method. We assume that the samples of PUs signals are independent zero-mean circular complex Gaussian distributed random variables in temporal
domain, but they might be correlated among different subbands. The additive noise samples are modeled as zero-mean random variables with circular complex Gaussian distribution that are independent in time and space but are not identical in different subbands. We assume that at least a given minimum number of subbands, i.e., $N$ out of $K$ subbands are vacant. In practice this number can be obtained using the history of PUs activity over the whole scanning spectrum.

The objective of wideband spectrum sensing is to detect the absence or presence of the PU signal in the $k^{th} \in \{1, 2, \cdots, K\}$ subband. Let $D \subseteq \{1, 2, \cdots, K\}$ be the subset of indexes of the occupied subbands. If the PU signal in the $k^{th}$ subband is absent, i.e., under hypothesis $H_0$, then $|D| = M$. If the PU signal in $k^{th}$ subband is present, i.e. under hypothesis $H_1$, then $|D| = M + 1$ where $M = K - N$ and $|.|$ is the cardinality of set ($\cdot$). We denote the covariance matrix of samples of PUs signals in the different subbands under hypothesis $H_0$ with $\Sigma_{S_0}$. This is a matrix of rank $M$ where the $k^{th}$ row and the $k^{th}$ column of $\Sigma_{S_0}$ is zero. On the other hand, $\Sigma_{S_1}$ denotes the covariance matrix of samples of PUs signals in the different subbands under hypothesis $H_1$. This is a matrix of rank $M + 1$ where the $k^{th}$ row and the $k^{th}$ column of $\Sigma_{S_1}$ that contain the variance of PU signal in the $k^{th}$ subband and the correlation of PU signal in the $k^{th}$ subband with other occupied subbands is nonzero. Thus we can consider the following binary hypothesis testing problem:

\[
\begin{align*}
H_0 : \quad Y &\sim CN(0, \Sigma_0) \quad \text{if the } k^{th} \text{ subband is vacant}, \\
H_1 : \quad Y &\sim CN(0, \Sigma_1) \quad \text{if the } k^{th} \text{ subband is occupied}
\end{align*}
\]

where $Y = [y_1, \cdots, y_L] \in \mathbb{C}^{K \times L}$ is the observation matrix. $\Sigma_0$ and $\Sigma_1$ denote the covariance matrices of each column of $Y$ under hypothesis $H_0$ and $H_1$, respectively. Therefore, we can write

\[
\begin{align*}
\Sigma_0 &= \Sigma_{S_0} + \Sigma_N, \\
\Sigma_1 &= \Sigma_{S_1} + \Sigma_N
\end{align*}
\]

where $\Sigma_N = \text{diag}\{\sigma_1^2, \cdots, \sigma_K^2\}$ is the covariance matrix of noise samples in the different subbands.

### III. Optimal Detector

For the optimal spectrum sensing, it is assumed that the matrices $\Sigma_{S_0}, \Sigma_{S_1}$ and $\Sigma_N$ or equivalently $\Sigma_0$ and $\Sigma_1$ are known. From (1), under hypothesis $H_0$, the Probability Density Function (PDF) of the observation matrix, $Y$, is as follows [10]

\[
f(Y \mid H_0, \Sigma_{S_0}, \Sigma_N) = \prod_{l=1}^{L} \frac{\exp\{-y_l^H \Sigma_0^{-1} y_l\}}{\pi^K |\Sigma_0|^L} = \frac{\exp\{-L \text{tr}(R \Sigma_0^{-1})\}}{\pi^{KL} |\Sigma_0|^L}. \tag{4}
\]

where $R \triangleq \frac{1}{L} YY^H$ is the sample covariance matrix and $\text{tr}(\cdot)$ denotes the trace of the matrix. By taking logarithm of (4), we have

\[
\begin{align*}
\mathcal{L}_0(Y) &= \ln f(Y \mid H_0, \Sigma_{S_0}, \Sigma_N) \\
&= -L \text{tr}(R \Sigma_0^{-1}) - L \ln |\det(\Sigma_0)| - KL \ln \pi. \tag{5}
\end{align*}
\]

Similarly from (1), under $H_1$, we obtain

\[
\begin{align*}
f(Y \mid H_1, \Sigma_{S_1}, \Sigma_N) &= \prod_{l=1}^{L} \frac{\exp\{-y_l^H \Sigma_1^{-1} y_l\}}{\pi^K |\Sigma_1|^L} = \frac{\exp\{-L \text{tr}(R \Sigma_1^{-1})\}}{\pi^{KL} |\Sigma_1|^L}. \tag{6}
\end{align*}
\]

By taking logarithm of (6), we have

\[
\begin{align*}
\mathcal{L}_1(Y) &= \ln f(Y \mid H_1, \Sigma_{S_1}, \Sigma_N) \\
&= -L \text{tr}(R \Sigma_1^{-1}) - L \ln |\det(\Sigma_1)| - KL \ln \pi. \tag{7}
\end{align*}
\]

From (5) and (7), the Logarithm of Likelihood Ratio (LLR) will be obtained as

\[
\begin{align*}
\text{LLR} &= \ln f(Y \mid H_1, \Sigma_{S_1}, \Sigma_N) - \ln f(Y \mid H_0, \Sigma_{S_0}, \Sigma_N) \\
&= L \text{tr}(R (\Sigma_1^{-1} - \Sigma_0^{-1})) + L \ln \frac{\det(\Sigma_0)}{\det(\Sigma_1)} \geq \tau_0. \tag{8}
\end{align*}
\]

By incorporating the constant terms in the decision threshold, (8) can be rewritten as

\[
T_{\text{opt}} = \frac{\text{tr}(R \Sigma_1)}{\Sigma_0} \geq \tau_0 \tag{9}
\]

where $\tau \triangleq \frac{1}{L} (\tau_0 + L \ln \frac{|\det(\Sigma_1)|}{|\det(\Sigma_0)|})$ and $\Sigma \triangleq \Sigma_0^{-1} - \Sigma_1^{-1}$.

### IV. GLR Detectors

In practical scenarios, the SU might not have any knowledge about some or all of $\Sigma_{S_0}, \Sigma_{S_1}$ and $\Sigma_N$. In such cases, as an alternative approach, we can use the GLR test to decide about the presence or absence of the PU signals. In this section, we derive the GLR tests for such practical scenarios.

#### A. Unknown PU Covariance Matrix $\Sigma_S$ (GLRDI)

In this part, we obtain the GLR detector when $\Sigma_{S_0}$ and $\Sigma_{S_1}$ are unknown but the covariance matrix of noise, i.e. $\Sigma_N$, is known. Using the matrix inversion lemma [11], (5) can be rewritten as

\[
\begin{align*}
\mathcal{L}_0(Y) &= -L \text{tr}(R \Sigma_N^{-1} (\Sigma_{S_0} \Sigma_N^{-1} + I)^{-1}) \\
&\quad - L \ln \left[ \det(\Sigma_{S_0} \Sigma_N^{-1} + I) \right] \\
&\quad - L \ln \left[ \det(\Sigma_N) \right] - KL \ln \pi. \tag{10}
\end{align*}
\]

If we define $E \triangleq R \Sigma_N^{-1}$ and $D_0 \triangleq \Sigma_{S_0} \Sigma_N^{-1}$, then from (10) we have

\[
\begin{align*}
\mathcal{L}_0(Y) &= -L \text{tr}(E(D_0 + I)^{-1}) - KL \ln \pi \\
&\quad - L \ln \left[ \det(D_0 + I) \right] - L \ln \left[ \det(\Sigma_N) \right]. \tag{11}
\end{align*}
\]
We note that the product of two matrices has the minimum of their ranks, so $D_0$ and $E$ are $M$-rank and $K$-rank matrices, respectively. Now by using the Singular Value Decomposition (SVD), we can write
\[ D_0 = S_0 \Theta_0 S_0^H \] (12)
with $\Theta_0 = \text{diag} \{ \theta_0, \ldots, \theta_0, 0, \ldots, 0 \}$ and $\theta_{0i} \geq \theta_{0j} \geq \ldots \geq \theta_{0M}$. Also, we have
\[ E = T \Psi T^H \] (13)
where $\Psi = \text{diag} \{ \psi_1, \ldots, \psi_K \}$ with $\psi_1 \geq \psi_2 \geq \ldots \geq \psi_K$. Thus (11) can be rewritten as
\[
L_0(Y) = -L \text{tr} \left( \Psi^H S_0 (\Theta_0 + I)^{-1} S_0^H T \right) \\
- L \ln \left[ \det (\Theta_0 + I) \right] \\
- L \ln \left[ \det (\Sigma_N) \right] - KL \ln \pi. 
\] (14)
The ML estimation of $S_0$ is given by
\[
\hat{S}_0 = \arg \max_{S_0 \in \mathbb{R}^K} L_0(Y) = \arg \min_{S_0 \in \mathbb{R}^K} \text{tr} \left( \Psi^H S_0 (\Theta_0 + I)^{-1} S_0^H T \right). 
\] (15)
Since $TS_0^H$ is a matrix with orthogonal vectors, $\Psi$ and $(\Theta_0 + I)^{-1}$ are diagonal matrices and (15) will be minimum if $TS_0^H = I_K$ [12]. Hence, we have
\[
\hat{S}_0 = T. 
\] (16)
By replacing (16) in (14), we obtain
\[
L_0(Y) = -L \sum_{i=1}^{M} \frac{\psi_i}{\theta_0 + 1} - L \sum_{i=M+1}^{K} \psi_i \\
- L \sum_{i=1}^{M} \ln \left[ \theta_0 + 1 \right] \\
- L \ln \left[ \det (\Sigma_N) \right] - KL \ln \pi. 
\] (17)
In order to obtain the ML estimation of $\Theta_0$, we need to compute
\[
\hat{\Theta}_0 = \arg \max_{\theta_0, \ldots, \theta_0, \in \mathbb{R}^+} L_0(Y). 
\] (18)
By setting $\frac{\partial}{\partial \theta_{0i}} L_0(Y) = 0$, it can be easily shown that
\[
\hat{\theta}_{0i} = \psi_i - 1; \quad i = 1, \ldots, M. 
\] (19)
By replacing the above estimation in (10), we obtain
\[
L_0(Y) = -LM - L \sum_{i=M+1}^{L} \psi_i - L \sum_{i=1}^{M} \ln \psi_i \\
- L \ln \left[ \det (\Sigma_N) \right] - KL \ln \pi. 
\] (20)
Similarly, for hypothesis $H_1$ we have
\[
L_1(Y) = -L \text{tr} \left( E (D_1 + I)^{-1} \right) - L \ln \left[ \det (D_1 + I) \right] \\
- L \ln \left[ \det (\Sigma_N) \right] - KL \ln \pi. 
\] (21)
where $D_1 \triangleq \Sigma_N^{-1}$ is a $(M+1)$-rank matrix and it can be expressed as
\[
D_1 = S_1 \Theta_1 S_1^H 
\] (22)
where $\Theta_1 = \text{diag} \{ \theta_1, \ldots, \theta_{1M+1}, 0, \ldots, 0 \}$ with $\theta_1 \geq \ldots \geq \theta_{1M+1}$. Thus similar to hypothesis $H_0$, we will obtain the ML estimation of $S_1$ and $\Theta_1$ as
\[
\begin{cases} 
\hat{S}_1 = T, \\
\hat{\theta}_{1i} = \psi_i - 1; \quad i = 1, \ldots, M + 1.
\end{cases} 
\] (23)
By substituting the above results in (21), we have
\[
L_1(Y) = -L(M + 1) - L \sum_{i=M+2}^{M+1} \psi_i \\
- L \sum_{i=1}^{M+1} \ln \psi_i - L \ln \left[ \det (\Sigma_N) \right] \\
- KL \ln \pi. 
\] (24)
From (20) and (24), the LLR function is obtained as
\[
\text{LLR} = \ln \frac{f(Y | H_1, \hat{S}_1, \Sigma_N)}{f(Y | H_0, \hat{S}_0, \Sigma_N)} = L_1(Y) - L_0(Y) \\
= L \psi_{M+1} - L \ln \psi_{M+1} - L \ln \left[ \frac{\psi_{M+1} \tau_i}{\hat{\psi}_{M+1}} \right], 
\] (25)
and therefor the GLR detector will be
\[
T_{GLR1} = \psi_{M+1} - \ln \psi_{M+1} \frac{\tau_i}{\hat{\psi}_{M+1}}. 
\] (26)
where $\tau = \frac{\sigma^2}{\sigma^2} + 1$ is the decision threshold and can be calculated for a given false alarm probability $P_{fa}$.

B. Unknown $\Sigma_S$ and $\Sigma_N$ (GLRD2)

In this part, we consider the case that the SU has no knowledge about $\Sigma_S$ and $\Sigma_N$. Under this condition, we first estimate $\Sigma_N$ whereas $\Sigma_S$ is assumed to be known a parameter and then obtain the ML estimate of $\Sigma_S$ whereas $\Sigma_N$ is known. Since the estimations of $D$ under both hypothesis $H_0$ and $H_1$ were calculated in Section IV-A, the estimation of $\Sigma_S$ under $H_0$ and $H_1$ can be easily computed for this case
$(\Sigma_S = D \Sigma_N$).
Now we assume that $\Sigma_S$ is known to obtain the estimation of covariance matrix of noise. Under $H_0$ and $H_1$, since $\Sigma_S$ is not a full rank matrix, it does not have inverse but we can rewrite (5) as
\[
L_0(Y) = -L \text{tr} \left( R (\Sigma_S + I + \Sigma_N - I)^{-1} \right) \\
- L \ln \left[ \det (\Sigma_S + I + \Sigma_N - I) \right] - KL \ln \pi. 
\] (27)
If we define $\Sigma_N - I \triangleq A$ and $\Sigma_S + I + \Sigma_N - I \triangleq B_0$, thus $B_0$ is a full rank matrix and therefore has a inverse, therefore (27) can be rewritten as
\[
L_0(Y) = -L \text{tr} \left( R B_0^{-1} (I + AB_0^{-1}) \right) \\
- L \ln \left[ \det (I + AB_0^{-1}) \right] - L \ln \left[ \det (B_0) \right] \\
- KL \ln \pi. 
\] (28)
Now we define \( W_0 = P_0 \Delta_0 P_0^H \); \( \Delta_0 \) is the diagonal \( \{ \delta_0_1, \cdots, \delta_0_K \} \) (29) with \( \delta_0_1 \geq \delta_0_2 \geq \cdots \geq \delta_0_K \) and
\[
Q_0 = G_0 \Omega_0 G_0^H \quad ; \quad \Omega_0 = \text{diag} \{ \omega_0_1, \cdots, \omega_0_K \}
\] (30) with \( \omega_0_1 \geq \omega_0_2 \geq \cdots \geq \omega_0_K \). So we have
\[
\mathcal{L}_0 (Y) = -L \text{tr} \left( \Delta_0 P_0^H G_0 (I + \Omega_0) \Delta_0 P_0^H G_0^H P_0 \right) - L \ln \left( \det (I + \Omega_0) \right) - L \ln \left( \det (B_0) \right) - KL \ln \pi.
\] (31)

It can be easily shown that
\[
\{ \begin{align*}
    G_0 & = P, \\
    \omega_i & = \delta_0_i - 1
\end{align*} \quad ; \quad i = 1, \cdots, M + 1.
\] (32)

Therefore, \( Q_0 = W_0 - I = RB_0^{-1} - I \). Since \( \Sigma_N = Q_0 B_0 + I \) and \( B_0 \) and \( I \) are known, we have
\[
\hat{\Sigma}_{N_0} = R - \Sigma_{S_0}.
\] (33)

Similarly, it can be easily shown that
\[
\hat{\Sigma}_{N_1} = R - \Sigma_{S_1}.
\] (34)

In order to obtain the LLR function when \( \Sigma_S \) and \( \Sigma_N \) are unknown, we can use an iterative algorithm provided in Algorithm I. The updated ML estimations of the unknown parameters are replaced in (8), so we have
\[
T_\text{GLRD2} = \ln \frac{f(Y \mid H_1, \hat{\Sigma}_{S_1}, \hat{\Sigma}_{N})}{f(Y \mid H_0, \hat{\Sigma}_{S_0}, \hat{\Sigma}_{N})}
= L \text{tr} \left( R (\hat{\Sigma}_0^{-1} - \hat{\Sigma}_1^{-1}) \right)
+ L \ln \frac{\det (\hat{\Sigma}_0)}{\det (\hat{\Sigma}_1)} \implies \tau_0.
\] (35)

Therefore the GLR test can be derived as
\[
T_\text{GLRD2} = \text{tr} \left( R (\hat{\Sigma}_0^{-1} - \hat{\Sigma}_1^{-1}) \right)
+ \ln \frac{\det (\hat{\Sigma}_0)}{\det (\hat{\Sigma}_1)} \implies \tau_0
\] (36)

where \( \tau \defeq \tau_0 / L \).

In each step of the employed iterative algorithm, we maximize the LLR function with respect to unknown parameters using the obtained estimated parameters in the previous step. Therefore, \( T_\text{GLRD2} \) which is based on the LLR function gets closer to its maximum value and hence the estimated values converge to their ML estimation. The accuracy of this algorithm in the estimation of unknown parameters depends on number of its iterations.

\textbf{Algorithm I: Iterative algorithm for estimation \( \Sigma_0, \Sigma_1 \) and \( \Sigma_N \).}

\begin{itemize}
    \item Initialize \( \Sigma_{(0)} = \Sigma_{(0)}^{(0)} \)
    \item Set \( i = 0 \)
    \item \textbf{Repeat}
        \begin{itemize}
            \item compute \( \hat{\Sigma}_{N_0}^{(i+1)} \) from (16) and (19)
            \item compute \( \hat{\Sigma}_{N_1}^{(i+1)} \) from (23)
            \item compute \( \hat{\Sigma}_{N_0}^{(i+1)} \) from (33)
            \item compute \( \hat{\Sigma}_{N_1}^{(i+1)} \) from (34)
        \end{itemize}
    \item \textbf{Until convergence}
\end{itemize}

\section{Analytical Performance Evaluation}

In this section, we evaluate the performance of the derived optimal detector and GLRD1 in terms of detection probability, \( P_d \), and false alarm probability, \( P_a \). This requires the calculation of Complementary Cumulative Distribution Function (CCDF) of the test statistics under \( H_0 \) and \( H_1 \). Since the derivation for the general case is mathematically intractable, we obtain them for a special case where SNR approaches to zero which is more useful and also likely in the practical scenarios. We note that since \( R \) is the ML estimation of \( \Sigma \) and \( \Sigma_N \) is a diagonal matrix, \( R \) would be approximately a diagonal matrix for low values of SNR. In addition, it is assumed that the number of PUs signal samples, \( L \), is large enough. Let \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K \) be the eigenvalues of \( R \). Also let \( \xi_{j_1} > \xi_{j_2} > \cdots > \xi_{j_K} \) and \( \xi_{j_1} > \xi_{j_2} > \cdots > \xi_{j_K} \) be the eigenvalues of \( \Sigma_0 \) and \( \Sigma_1 \), respectively. Considering these assumptions we have the following Lemma [13]:

\paragraph{Lemma:} For \( L \rightarrow \infty \), under both hypothesis we have
\[
\sqrt{L} \left( \frac{\lambda_j}{\xi_{j_i}} - 1 \right) \sim N(0, 1),
\] (37)

for \( j = 0, 1 \) and \( i = 1, 2, \cdots, K \). Furthermore \( \sqrt{L} \left( \frac{\lambda_k}{\xi_{j_k}} - 1 \right) \) is independent of \( \sqrt{L} \left( \frac{\lambda_i}{\xi_{j_i}} - 1 \right) \) for \( i \neq k \).

In the following, we use this Lemma to derive the performance of detectors.

\subsection{The performance of optimal detector}

Under our assumptions above, (9) can be approximated as
\[
T_{\text{opt}} \simeq \sum_{i=1}^{K} \Sigma_{i,i} \lambda_i \xi_{j_i} \implies \tau \quad \text{opt},
\] (38)

where \( \Sigma_{i,i} \) denotes the \( i^{th} \) element of main diagonal of \( \Sigma \). After some mathematical manipulations, under \( H_0 \), (38) yields
\[
\sum_{i=1}^{K} \Sigma_{i,i} \sqrt{L} \left( \frac{\lambda_i}{\xi_{j_i}} - 1 \right) \implies \sqrt{L} \left( \frac{\tau}{\det(\Sigma_0)} - \sum_{i=1}^{K} \frac{\Sigma_{i,i}}{\prod_{j \neq i} (\xi_{j})} \right).
\] (39)
Therefore from Lemma, $P_{fa}$ can be easily obtained as

$$P_{fa} = P[T_{opt} > \tau \mid \mathcal{H}_0]$$

$$= Q\left(\frac{\sqrt{L(\tau_{det}(\Sigma_0)) - \sum_{i=1}^{K} \frac{\Sigma_{i,i}}{\prod_{j \neq i}^{K} \xi_{j}^2}}}{\sqrt{\sum_{i=1}^{K} \left(\frac{\Sigma_{i,i}}{\prod_{j \neq i}^{K} \xi_{j}^2}\right)^2}}\right)$$

(40)

where $Q(x) = \int_{x}^{\infty} e^{-t^2}dt$ is GAUSSIAN Q function. Similarly, for detection probability, we obtain

$$P_{d} = P[T_{opt} > \tau \mid \mathcal{H}_1]$$

$$= Q\left(\frac{\sqrt{L(\tau_{det}(\Sigma_1)) - \sum_{i=1}^{K} \frac{\Sigma_{i,i}}{\prod_{j \neq i}^{K} \xi_{j}^2}}}{\sqrt{\sum_{i=1}^{K} \left(\frac{\Sigma_{i,i}}{\prod_{j \neq i}^{K} \xi_{j}^2}\right)^2}}\right)$$

(41)

### B. Performance of GLRD1

If SNR approaches to zero, $\Sigma_{S_0}$ and $\Sigma_{S_1}$ will be diagonal matrices and it can be easily shown that the eigenvalues of $\Sigma$ will be $\frac{\lambda_1}{\sigma^2_1}, \frac{\lambda_2}{\sigma^2_2}, \ldots, \frac{\lambda_K}{\sigma^2_K}$. Thus (26) can be rewritten as:

$$T_{GLRD1} \simeq \frac{\lambda_{M+1}}{\sigma^2_{M+1}} - \ln \left[\frac{\lambda_{M+1}}{\sigma^2_{M+1}}\right] \mathcal{H}_1 \geq \mathcal{H}_0 \geq \tau.$$  

(42)

Simulation results show that $\frac{\lambda_{M+1}}{\sigma^2_{M+1}}$ with high probability is greater than one. Noting that $h(x) = x - \ln x$ for $x > 1$ is a monotonically increasing function, the above detector can be simplified as

$$T_{GLRD1} \simeq \frac{\lambda_{M+1}}{\sigma^2_{M+1}} \mathcal{H}_1 \geq \mathcal{H}_0.$$  

(43)

From Lemma, we have

$$P_{fa} = P[T_{GLRD1} > \eta \mid \mathcal{H}_0]$$

$$= Q\left(\sqrt{L(\eta - 1)}\right).$$  

(44)

Let $\rho_{1} \geq \cdots \geq \rho_1(M+1) \geq 0 = \cdots = 0$ be the eigenvalues of $\Sigma_{S_1}$. We can easily show that

$$P_{d} = P[T_{GLRD1} > \eta \mid \mathcal{H}_1]$$

$$= Q\left(\sqrt{L\left(\frac{\eta}{1 + \frac{\rho_{1}(M+1)}{\sqrt{\sigma_{M+1}^2}} - 1}\right)}\right).$$  

(45)

We note that in (45), $\frac{\rho_{1}(M+1)}{\sqrt{\sigma_{M+1}^2}}$ is function of two parameters: the SNR of the $M+1^{th}$ subband and correlation of $M+1^{th}$ subband with the other subbands. Now by increasing the SNR or decreasing the correlation of the $M+1^{th}$ subband with other subbands, the $P_{d}$ will increase and conforms with our expectation.

### VI. SIMULATION RESULTS

In this section, we present some simulation results to evaluate and compare the performance of the optimal and proposed GLR detectors. We have used the Monte-Carlo method for simulation by $10^5$ independent runs. Under $\mathcal{H}_0$, we have randomly generated the decision statistic considering its distribution for each false alarm probability. Then, we have calculated the detection threshold so that $100P_{fa}$ percentile of generated data are above the determined threshold. We define the average SNR respectively under hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$ as

$$\mathcal{H}_0: \bar{\gamma} \triangleq \frac{\text{tr}(\Sigma_{S_0})}{M\sigma_{n_0}^2}$$

(46)

$$\mathcal{H}_1: \bar{\gamma} \triangleq \frac{\text{tr}(\Sigma_{S_1})}{(M+1)\sigma_{n_1}^2}$$

(47)

where $\sigma_{n_0}^2$ and $\sigma_{n_1}^2$ are the arithmetic mean of the noise variances in the occupied subbands respectively under hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$. In Figure 1, we compare the performance of the proposed GLR detectors with the optimal detector versus SNR for false alarm probability of $P_{fa} = 10^{-2}$, $L = 50$, $K = 14$ and $M = 8$. As expected, the GLRD1 (which is assumed that have the exact knowledge of the noise covariance matrix) outperforms GLRD2.

Figure 2 shows the detection probability $P_{d}$ versus the probability of false alarm $P_{fa}$ i.e., complementary ROC for $M = 8$, $\bar{\gamma} = 7$ dB and for different values of $L$ and $K$ when their product is fixed and equals to 600, i.e., $KL = 600$. We consider a fixed value for $KL$ to investigate the effect of number of samples versus the number of subbands. As it can be observed from Figure 2, the effect of the number of temporal samples in each subband, i.e., $L$, on the performance improvement is more than the number of subbands $K$ for
Thus in practice, we have to make a trade-off between a constant number of occupied subbands $M$. Unfortunately, we can not increase $L$ arbitrarily since $L$ determines the acquisition time (the waiting time-lag before a decision can be made). Thus in practice, we have to make a trade-off between $P_{fa}$ (the spectrum usage efficiency), $P_{in}$ (PU interference protection level) and $L$ (the acquisition time).

In Figure 3 we illustrate the effect of $M$ for constant values of $K$ and $L$ assuming on the average SNR of $7$ dB. As can be observed from this figure, for the fixed values of $K$ and $L$ by decreasing value of $M$ the performance of detectors improves. This is expected since under the condition of the correlated subbands available knowledge on the occupied subbands increase and results in better spectrum sensing.

VII. CONCLUSION

In this paper, we have studied the problem of spectrum sensing for a wideband spectrum divided into multiple subbands. Assuming that PUs signals are correlated in the different subbands, we have proposed GLR detectors for two cases a) the covariance matrix of PUs signals is unknown and b) the PU covariance matrix and noise variance are both unknown. We have analytically evaluated the false alarm and detection probabilities for the proposed detectors and compared their performance.

REFERENCES